the dispersive regime. It would appear that a more detailed examination of these theories is needed before any of their results can be used as a standard for comparison when $2l/t \simeq 1$.

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Accurate Solution of Microstrip and Coplanar Structures for Dispersion and for Dielectric and Conductor Losses

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***Abstract*—For the analysis of coplanar- and microstrip-type structures, a higher order solution of the spectral-domain approach is introduced. Legendre polynomials are used as the basis functions for fields having singularities near the edges, leading to fast convergence to the exact solution. A perturbation technique is combined with the spectral-domain method to evaluate conductor and dielectric losses in microstrip, inverted microstrip, and coupled microstrip in the metallic enclosure. Computations of characteristic impedance and losses incurred in several structures are also presented. Central processing unit (CPU) time on an IBM 360/65 for the zeroth-order approximation ranges from 1 to about 5 s for the whole computation, and increases if higher order of solution is requested for better accuracy. The calculation of attenuation due to conductor losses is found to be particularly sensitive to order of approximation, so that the generally used "zeroth-order" solution is inadequate. A user-oriented program package has been written, including options on order of mode, order of solution (i.e., of approximation), impedance, attenuation, and number of substrates. Although written for single or coupled microstrip, the program can be adapted for arbitrary arrangements of thin coplanar conductors. The program is described separately.**

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I. Introduction

THE WIDESPREAD use of MIC's in recent years has caused rapid progress in the theory and technology of it. The very first transmission line used in MIC was, indeed, microstrip laid on the dielectric substrate, and then other transmission lines such as slot line, suspended microstrip, and so on, were introduced and improved.

Initially, the analysis for this class of transmission line was invariably a quasi-TEM approximation, except for slot line where Cohn [1] introduced a frequency dependent solution because of its different nature. Although a quasi-TEM solution at low frequency can yield satisfactory results, at high frequency its weakness becomes apparent. To feature the frequency dependence of these lines, one must consider a hybrid mode analysis which in turn is more tedious, and in some cases requires enormous computing time. This dispersion analysis was studied by various workers and by various methods. For instance,

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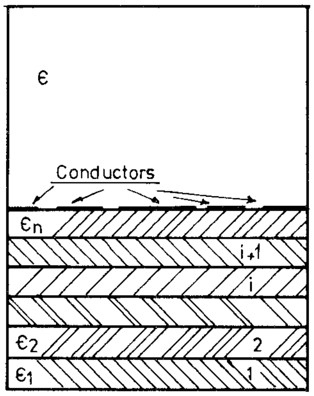


Fig. 1. Shielded multilayer dielectric with arbitrary coplanar conductors.

Mittra and Itoh [2] solved shielded microstrip problems via an integral equation, and Denlinger [3] cast the open version of it into the spectral domain and then solved the transcendental equations by means of a coupled integral-equation method.

In 1973 and 1974, Itoh and Mittra [4], [5] used a spectral-domain approach together with Galerkin's method to solve both shielded and unshielded microstrip. This combination reduced computing time drastically in contrast to the other (integral-equation) methods which required much computing time, as well as memory. This theory was developed and modified for multilayers and multiconductor structures by the authors [6] where a transfer matrix fulfills all the boundary conditions between successive dielectric layers. This approach has now been extended further by the authors, by including and studying 1) the calculation of attenuation due to dielectric and conductor loss, 2) calculation of characteristic impedance, 3) inclusion of Legendre polynomials, as well as trigonometric basis functions for the field or current density, 4) convergence for various orders of matrix and number of basis functions, and 5) consideration of higher order modes, as well as the cutoff free mode. To amplify on 4) above, the method converges rapidly to the "exact" solution of the zero thickness, perfectly conducting structure of Fig. 1. Subject only to small tan δ of the dielectric, and high conductivity of the metal, calculation of losses are also "exact in the limit", and do not involve empirical approximations (as, for instance, in Jansen [20]).

As part of this study, a user-orientated program package has been developed and is described elsewhere [21] for the above calculation, with options on various coplanar conductors on singular or multilayer substrates. Results have been checked against most available structures such as slot line, coupled microstrip, suspended or shielded inverted microstrip, and so on. The importance of the losses in the MIC is a significant point which can be quantified by this method. Dielectric and conductor losses are both investigated through a perturbation formulation. Conductor losses are found to be particularly sensitive to choice of basis function, and some earlier theories are found to be inadequate.

II. Theory

As the finite spectral-domain theory has been described for shielded microstrip [5] and generalized for shielded multilayer multiconductor structures [6], just a brief review is given here. Fig. 1 shows a generic cross section of multilayer and multiconductor structure. Potential functions and, consequently, fields in each layer can be written as follows:

$$E_{zi}=j\frac{(k_i^2-\beta^2)}{\beta}\psi_i^e(x,y)e^{-j\beta z}$$

$$H_{zi}=j\frac{(k_i^2-\beta^2)}{\beta}\psi_i^h(x,y)e^{-j\beta z},\qquad i=1,2,\cdots,n+1. \quad (1)$$

Casting the above fields in the spectral domain, by finite Fourier transform in the x direction, and satisfying boundary conditions at the interfaces of the consecutive layers, leads to the following matrix form:

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}\begin{bmatrix} \tilde{J}_x \\ \tilde{J}_z \end{bmatrix}=\begin{bmatrix} \tilde{E}_z \\ \tilde{E}_x \end{bmatrix} \quad (2)$$

where tilda (˜) specifies fields or currents in the spectral domain. A dual form of the above matrix representation is given by

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} \tilde{E}_z \\ \tilde{E}_x \end{bmatrix}=\begin{bmatrix} \tilde{J}_x \\ \tilde{J}_z \end{bmatrix}. \quad (3)$$

$\tilde{E}_s$ and $\tilde{J}_s$ used in both matrixes are transforms of the electric field and current density at the interface where the conductors are laid. Whether it is advantageous to use (2) or (3) depends upon the type of conductor arrangement; for instance, in the shielded microstrip case, (2) is preferred and in the case of shielded slot, (3). Both matrix forms can be employed for any problem, but accurate results given by one of them may be obtained easier and faster. This matter will be clarified later on in the script.

Consider (3) and let E_z and E_x be expressed in terms of a complete set of basis functions, truncated to

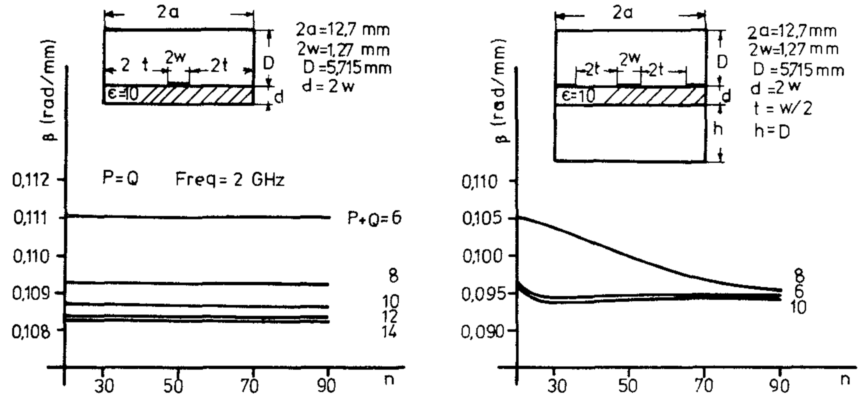
$$E_x=\sum_{m=1}^{P} a_m E_{xm}(x)$$

$$E_z=\sum_{m=1}^{Q} b_m E_{zm}(x). \quad (4)$$

After transformation of E_x and E_z into the spectral domain, substituting into (3) and using Galerkin's method as well as Parseval's theorem, the result is a set of $P+Q$ homogeneous simultaneous equations with $P+Q$ unknown, whose nontrivial solution yields the propagation constant of the structure.

A. Basis Functions and Convergence

To approximate either fields or currents across the interface where conductors are laid, a complete set of functions is required. Legendre polynomials are used for those having unbounded behavior near edges, while trigo-



(a) (b)

Fig. 2. Dependence of propagation constant on matrix order $(P+Q)$ and number of Fourier terms (n).

nometric functions are employed for bounded fields. Both functions have Fourier transforms in closed form. Use of Legendre polynomials may be justified if the generating function is considered, which is given by

$$(1-2t\,x+t^2)^{-1/2}=\sum_{n=0}^{\infty} P_n(x)\,t^n$$

and in the limit $t \to 1$:

$$\{2(1-x)\}^{-1/2}=\sum_{n=0}^{\infty} P_n(x) \tag{5}$$

where the left side indicates a singularity similar to that occurring in this type of problem [7]. Therefore, by choosing Legendre polynomials it is expected to have a fast convergence to the related fields. To test the convergence, two different problems are treated; the first one is microstrip and the second is shielded coplanar waveguide. From this discussion and the consideration of Fig. 2, the following expansions are valid:

$$E_x=\sum_{m=1}^{P} a_m P_{m-1}\{(x-w-t)/t\}$$
$$E_z=\sum_{m=1}^{Q} b_m \sin\{m\pi(x-w)/2t\}, \qquad w<|x|<w+2t. \tag{6}$$

The rest of the spectral-domain solution continues as described above in the text. A computer program implementing the calculation has been developed, and the results are depicted in Fig. 2(a) and (b) for various orders of matrix (order $=P+Q$, $P=Q$), and number of Fourier terms. Fig. 2(a) shows how the results improve by increasing the number of Legendre polynomials and virtually complete convergence is obtained by introducing a matrix of order 8 if an error of 1 percent or better is acceptable. From Fig. 2(b), where curves for shielded coplanar waveguide are sketched, we see the convergence with increasing order of matrix and number of Fourier terms. Solutions have also been obtained with $P \neq Q$ and sometimes this is advantageous, such as for very accurate results with microstrip at low frequencies. It is found that in structures where the field or current is approximated over a small distance, a good solution can be achieved by a low order of matrix, or even the zeroth-order solution [5], [6]. Consequently, these results highlight the advantages of either (2) or (3) in relevant problems. Results have been checked against [5], [6], and [8], always with very good agreement. Central processing unit (CPU) time clearly depends on the number of regions, order of matrix and number of Fourier terms. Usually for a matrix of order 8, CPU time will not exceed 5 s per point.

B. Higher Order Modes

At a high frequency of operation, existence of an enclosure causes propagation of higher order modes. To show the capability of the spectral-domain approach in analyzing these modes, shielded microstrip is brought under examination, and results are compared with those issued by Yamashita [8] and Mittra [2].

Comparison with Yamashita (who employed the integral-equation method with nonuniform discretization) gave agreement within the readability of the quoted results. Mittra used almost the same techniques, but near the end of the analysis he casts the obtained equations into a new form, (i.e., into "auxiliary" equations) where in turn, this new set of equations gives the opportunity of rapid convergence due to the asymptotic decrease of their coefficients.

Between the dominant and the first high-order modes of Fig. 3, Mittra [2] gives an additional mode. Our spectral-domain program also gives a solution close to Mittra's, but study of the associated electric field across the air–dielectric interface shows it to be a spurious nonphysical solution. Ganguly [22] also refers to the occurrence of spurious roots. It should be pointed out that in the results of Fig. 3, our investigation of higher order modes has been accomplished by using (3) and a 10×10 matrix, (i.e., fifth-order approximation, $P=Q=5$). Evidently the zeroth-order solution, due to lack of either J_x or E_z, cannot give accurate and reliable results.

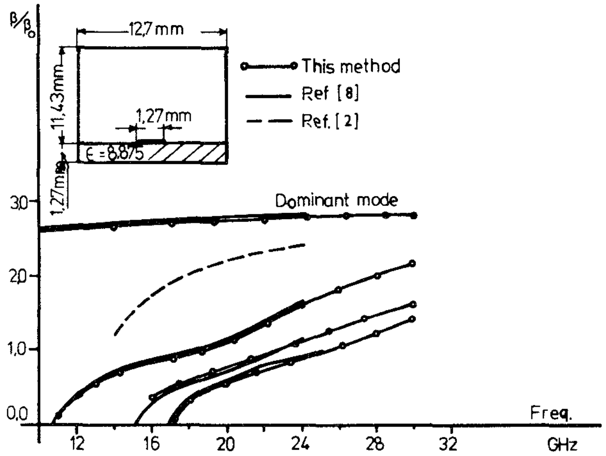


Fig. 3. Higher order modes of shielded microstrip.

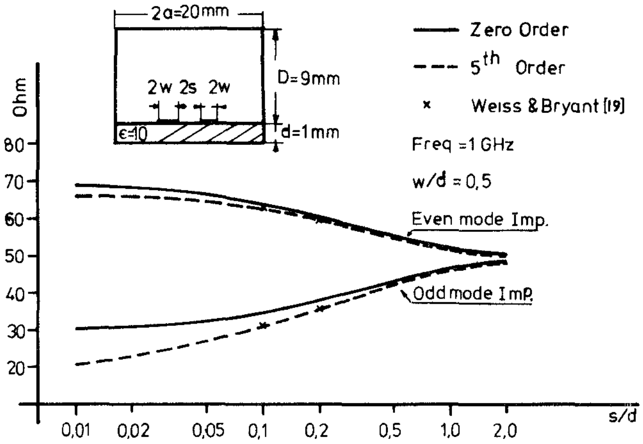


Fig. 4. Even- and odd-mode impedances of shielded coupled microstrips.

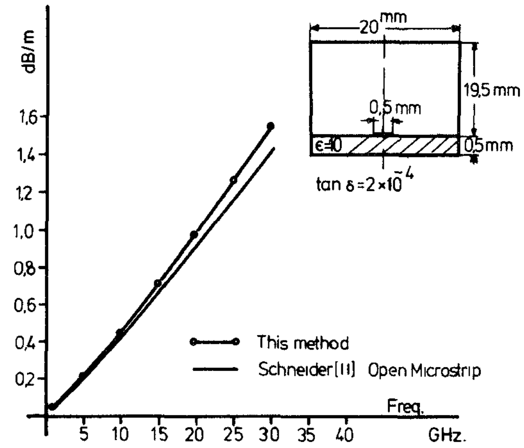


Fig. 5. Dielectric losses of shielded microstrip versus frequency.

C. Characteristic Impedance

Since the wave propagation is of hybrid-mode type in these structures, a precise and convenient definition of characteristic impedance is difficult. For instance, in normal microstrip, the total conduction currents in the two conductors are not even equal! However, a tentative definition of characteristic impedance used by other investigators is employed here, and the values are computed. As examples, the impedances of both shielded and coupled microstrips have been obtained by the following definitions:

$$Z_0 = \frac{2\ \text{Power}}{I_z^2}, \quad \text{for microstrip}$$

$$Z_0 = \frac{\text{Power}}{I_z^2}, \quad \text{for coupled microstrip} \tag{7}$$

where

$I_z = \int_{-w}^{+w} J_z(x)\,dx$, integration being over a complete strip.

Fig. 4 shows the results for shielded coupled microstrip in which the surrounding walls are chosen, such that they approximate well to an open structure. In this sketch solid lines indicate the zeroth-order solution, while dotted lines present the higher order of estimation, (fifth order). The current distribution for zeroth-order approximation was assumed of $(1/2w)(1+|x/w|^3)$ [9], [5]. As seen from Fig. 4, the accuracy of the zeroth-order solution depends on the separation of the two conductors, which means that with tight coupling, a zeroth-order solution should be treated cautiously. The frequency dependence of impedances has also been investigated and compared with results published by Krage and Haddad [10], and an agreement within 1 percent or better has been obtained.

D. Attenuation

To evaluate attenuation due to dielectric and conductor losses, the following methods are used.

Dielectric losses: Perturbation theory is used, based on 1) the loss tangent being sufficiently small, and 2) fields for the lossless condition, which are first computed, being used as the perturbed fields. To find an expression for α_d, Maxwell's equations are written for both lossy and lossless conditions, and then by applying Green's theorem to the integrals the following formula is obtained:

$$\alpha_d = \frac{\omega\epsilon \tan\delta \int_{S_{\text{diel}}} |E_0|^2 dS}{2\,\text{Re} \int_S E_0 \times H_0^* \cdot \vec{a}_z\, dS} \tag{8}$$

where S_{diel} is the area covered by dielectric, and S is the complete cross section. Subscript zero denotes the unperturbed fields. The above formulation was implemented in the computer program yielding impedances. As a comparison, shielded microstrip and shielded inverted microstrip are computed, and the side walls are located such that their effect becomes negligible. Figs. 5 and 6(a) give results for these two structures, and fairly good agreement is apparent with the available data [11], [12]. The advantage of this method over the two other (quasi-TEM) approximations can be noticed in the numerator of (8) where an extra term with E_z has been included, and contributed particularly at high frequencies, as in Fig. 5.

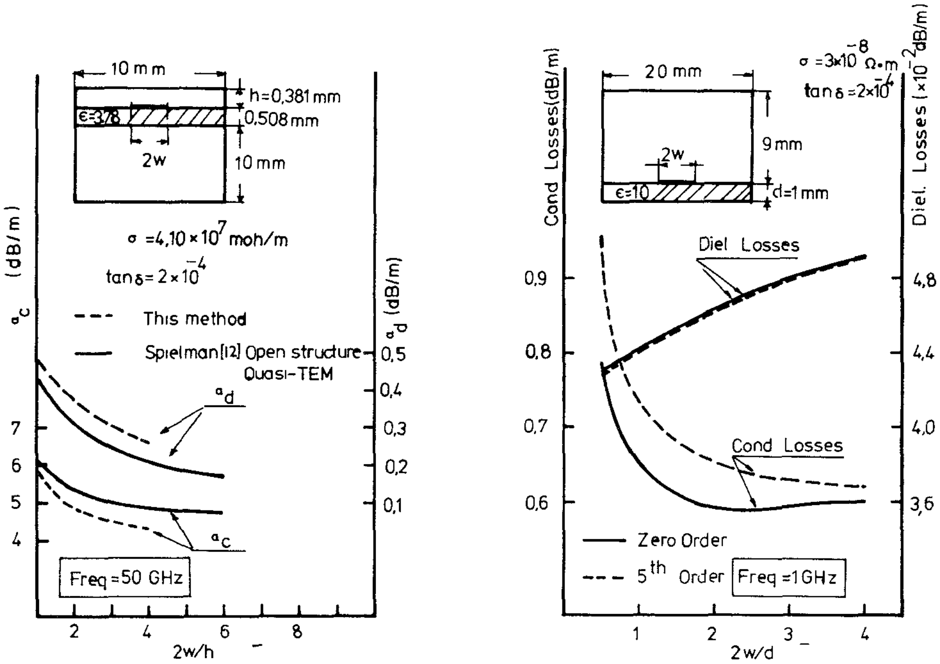


Fig. 6(a). Dielectric and conductor losses of shielded inverted microstrip for different linewidth. (b). Dielectric and conductor losses of shielded microstrip for different linewidth.

Fig. 7 exhibits the dielectric losses incurred in coupled microstrip lines and also the accuracy of the two different order of solutions.

Conductor losses: Attenuation due to the imperfect conductors is obtained via a conventional perturbation formula, viz.,

$$\alpha_c = \frac{R_s \int_C |H_t|^2 dl}{2 \operatorname{Re} \int_S E_0 \times H_0^* \cdot \vec{a}_z dS} \tag{9}$$

where R_s is the surface resistance [12], and $|H_t|$ is the amplitude of magnetic field at the conducting surfaces for the lossless case. The required fields are derived by the spectral-domain method and substituted in the expression for α_c and α_d. Conductor losses of microstrip, coupled microstrip, and inverted microstrip have been computed and depicted in Figs. 6(b), 7, and 6(a), respectively. Although H fields are calculated for an infinitely thin strip, it would be a good approximation to the case, where $t \ll h$, (t is the thickness of strip, and h is the height of dielectric) and also the integration of $|H_t|$ around the strip is valid when $t \gg \delta$, where δ is the skin depth. As it appears from Fig. 6(b), conductor losses are very sensitive to the order of solution. This could be attributed to the more accurate approximation of the fields by higher order solution, particularly near to the edges, which is very important for the conductor loss integral. Comparison of the result by this method (Fig. 6(a)) and those published very recently for inverted microstrip shows a discrepancy of up to 12 percent. This discrepancy can be attributed to two facts. First, due to the overestimation described in [12], and second, because of the additional side walls used in this theory. For $w/h = 1$, these walls are sufficiently distant to have negligible effect, but certainly they can affect the fields when $w/h = 4$. Conductor losses for microstrip have also been investigated and compared with those given by Schneider [13] and Pucel [14] who followed the same theory presented by Wheeler [15]. In both cases lower values of conductor losses are now predicted, and it appears that their methods overestimated the conductor losses, as in fact observed by [16] and [17], especially at high impedance lines. Fig. 7 shows the conducting losses

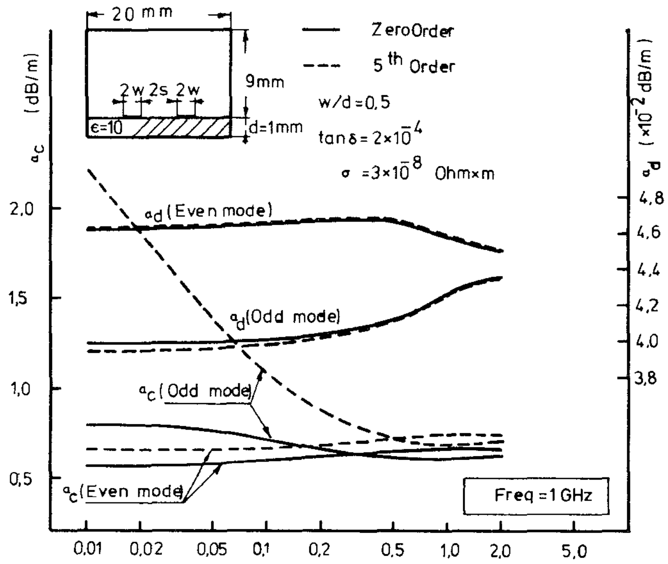


Fig. 7. Dielectric and conductor losses of shielded coupled microstrip versus line separation.

incurred in the coupled microstrip lines. It is evident that due to high concentration of the fields around the gap in the tight coupling for the odd mode, losses should be larger than that for the even mode, and it decreases as the strips are laid more apart. In one particular separation of the strips, losses for even and odd modes become equal; with increasing separation odd-mode losses become less than even-mode losses, and at the limit when they are far enough apart, they both have the same value which is equal to microstrip losses. This phenomenon has also been noticed in [18] where it refers to the lower value of dielectric losses in the odd-mode case which causes the intersection of the two curves at a smaller separation.

III. Conclusion:

A higher order solution of the spectral-domain analysis for shielded planar structures has been studied by the introduction of Legendre polynomials and trigonometric functions as basis functions. It is found that the correct choice gives faster convergence, and that more accurate and reliable solutions are indeed obtained by increasing the order of matrix and number of Fourier terms (more Fourier terms leading to better satisfaction of Parseval's identity). Higher order modes of microstrip, when E_z is even, were also sought and compared with the other available sources. Characteristic impedance and losses of some structures were computed, and it was found that although the zeroth-order solution can be useful for many practical structures, more precision is achieved by higher order of solution and is specially necessary for conductor losses, where a poor estimation of fields near the edges can lead to serious errors. The availability of the higher order solution, (as in the program package) clearly allows one to check the accuracy of the zeroth-order solution.

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